Motion of Particles Entrained in a Plasma Jet

The motion of small particles (glass microspheres, 30 to 140 microns in diameter) entrained in a free argon plasma jet was studied by means of high-speed cine streak photography. Radial temperature and velocity profiles as well as axial profiles of temperature, velocity, and argon concentration in the jet were experimentally determined by means of a plasma calorimetric probe. The system was found to be characterized by low relative Reynolds numbers (0.2 to 20) and extremely high deceleration rates (about −2,000 g). Under these conditions, an increase of drag coefficient over that predicted by the standard curve was experimentally observed. This increase was attributed to the nonsteady flow field around the particle (the so-called "history term" in the equation of motion). A general computer program has been proposed which predicts the particle velocity, acceleration and temperature along its trajectory.

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SCOPE

Plasma discharge devices operating at atmospheric pressure are now being used on an industrial basis as convenient sources of heat in high temperature reactors. The use of plasmas allows reliable continuous generation of gas temperatures in excess of 10,000 K with only minor restrictions on the composition of the gas. Industrial exploitation of such high energy levels is now being centered on chemical-metallurgical processes using multiparticle systems. Sayce (1971) has recently described most of the commercial devices presently available and has also identified a number of potentially promising metallurgical operations to which they could be applied. A specific example of a commercially successful process is the dissociation of zircon sands into zirconia and silica (Thorpe, 1971). Titanium dioxide has for several years been manufactured by the combustion of titanium tetrachloride in an atmosphere of oxygen heated by a radio-frequency (4 MHz) discharge device (Dundas and Thorpe, 1970). A number of other applications have recently been reviewed by Waldie (1972a).

The design of these plasma reactors requires a broad understanding of the transport processes and the reaction kinetics of high-temperature multiparticle systems. The study reported here is concerned with the momentum transfer between a free plasma jet (and its resulting tailflame) and entrained solid particles, with the overall aim of permitting the prediction of particle residence times. A knowledge of the latter is of paramount importance for design purposes since the total time of residence in the hot zone available for the heating of the particles and their complete conversion is generally a few milliseconds.

Reviews of the motion of particles entrained by isothermal gas streams have recently been published by Clift and Gauvin (1970, 1971). However, from a fluid-dynamics point of view, a particle entrained in a plasma jet is in an unusual regime:

1. The particle Reynolds number (0.2-20) is between the Stokesian ($Re < 0.1$) and boundary layer ($Re > 400$) ranges.
2. The flow field around the particle is highly nonisothermal with possible temperature ratios as high as 10 between free-stream and particle surface.
3. The flow field around the particle departs markedly from its steady state configuration due to the rapidly changing relative velocity between particle and gas, which can cause significant changes in the particle drag coefficient.

Specifically, this work reports on a comparison between the prediction and experimental measurement of the velocity and deceleration of particles emerging from a d.c. argon plasma flame, in an attempt to account for the unusual features listed above. The size range of particles used (30 to 140 μm) was chosen to coincide with the expected range of future commercial practice.

CONCLUSIONS AND SIGNIFICANCE

Experimental data on the flow characteristics of plasma flames and plasma tail flames in the published literature are very scanty. Even scarcer are data on the behavior and transport processes of particles entrained in such media. One of the major objectives of this study was therefore to provide such data. Shown in Figure 2 is the temperature radial profile of the argon jet obtained by means of a new plasma calorimetric probe at a fixed distance (178 mm) from the torch nozzle exit, while Figure 3 shows its radial velocity profile at the same location. Finally, axial profiles of temperature, velocity, and argon concentrations in the jet are shown in Figure 4.

The mean particle velocities of five separate fractions of closely-sized glass microspheres (in the overall size range 30 to 140 μm) were experimentally measured at a distance of 178 mm from the torch exit by means of a high-speed ciné camera operating in streak mode. From these data, an instantaneous value of the experimental coefficient of drag was determined. The latter was then compared with the value reported by Beard and Pruppacher (1969) under isothermal conditions in the N_{Re} range of interest (thus accounting for the low particle Reynolds number effect noted above), corrected by a kinematic viscosity ratio (to account for the nonisothermal effects). This comparison indicated that the experimentally determined coefficient of drag was larger than the calculated value. The excess drag was ascribed to the nonsteady nature of the flow field around the particle.

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A simulation study of the motion of the particles was then undertaken based on the Basset-Boussinesq-Oseen equation of motion. It will be remembered that in this equation the Basset history term accounts for the nonsteady motion. The distribution of temperature velocity, and concentration of the argon jet were modeled in the axial direction using the data experimentally obtained. A general computer program was developed incorporating corrections for nonisothermal effects and ionization and recombination effects. A set of computations was carried out for the mean particle diameter of each of the size ranges used. In addition, a set of computations was made with the history term neglected.

The results of the simulation indicated that, for each of the particle diameters investigated, the experimental values of the particle axial velocity fell between the predictions with and without history. It can therefore be concluded that the Basset term should be included in the equation of motion, but that it should be corrected by a coefficient as suggested by Odar and Hamilton (1964) and by Odar (1966).

The following general conclusions can be drawn from this work:

1. The measured velocities of particles of various diameters entrained by an argon plasma jet were in good agreement with the predictions of the computer program in which the nonisothermal, nonrelaxed nature of the flow field were taken into account.

2. Significant increases in the instantaneous drag on particles in a nonsteady system such as a plasma jet were predicted, ranging from about 20% for 30-μm particles to 100% for 150-μm particles. Experimental determination of the excess drag for the 44 to 53 μm particles were in agreement with these predictions.

3. Until more information is available, it is recommended that Odar's coefficient of 0.48 be used in connection with the Basset term in the equation of motion, when predicting the motion of particles in nonsteady systems.

The preliminary nature of the present study should be emphasized. Many aspects of the system require close examination. Chief among these are the rate of heat transfer to particles at low Reynolds numbers and under nonisothermal conditions and the turbulence characteristics of the system. The latter, in turn, are bound to affect the rate of heat transfer. The dependency of Odar's coefficient on particle diameter and other system parameters should also be determined. In spite of the unusual experimental difficulties inherent to this kind of investigation, it is hoped that these preliminary data will stimulate other workers in the field to refine the present findings.

PREVIOUS WORK

The basic equation for the drag force on a spherical particle at steady state in an isothermal system may be written

$$F_D = (-1/2)(C_D)(\rho_g)(\pi D_p^2/4)\,|V_p - V_g|\,(V_p - V_g) \tag{1}$$

Steady state isothermal determinations of drag coefficient (C_D) were carried out by Beard and Pruppacher (1969) in the range of interest for this study and confirmed by the numerical computations of Hamielec et al. (1967). These results fall a few percent above the standard drag curve (Lapple and Shepherd, 1940), but in view of the agreement between computation and experiments, the Beard and Pruppacher results are considered to be the most reliable data available:

$$0.2 \le N_{Re} \le 2, \qquad C_D = (24/N_{Re})(1 + 0.1\ N_{Re}^{0.99}) \tag{2a}$$

$$2 \le N_{Re} \le 21, \qquad C_D = (24/N_{Re})(1 + 0.11\ N_{Re}^{0.81}) \tag{2b}$$

$$21 \le N_{Re} \le 200 \qquad C_D = (24/N_{Re})\ (1 + 0.189\,N_{Re}^{0.63}) \tag{2c}$$

Rewriting Equation (1) for constant relative velocity in the form

$$F_D = K(\rho_g C_D) \tag{3}$$

where

$$K = (-1/2)(\pi D_p^2/4)\,|V_p - V_g|\,(V_p - V_g) \tag{4}$$

it is seen that both the terms inside the bracket of Equation (3) are functions of temperature, while K is independent of it. Any discussion of the effect of temperature on particle motion should therefore consider the product of the density and drag coefficient.

Lemoine and LeGoff (1969) have stated that the product ($\rho_g C_D$) is practically independent of temperature and therefore at any given relative velocity, the gas temperature is unimportant in determining particle acceleration. Their statement may be quite valid for different isothermal systems but cannot be applied to nonisothermal systems where there is a considerable temperature difference between particle surface and free-stream, as pointed out by Seymour (1971).

Kassoy et al. (1966) carried out numerical computations in the Stokesian regime and obtained a drag coefficient correction factor based on the free-stream properties of the gas, assuming a linear variation of fluid properties with temperature. Their results were approximated by the linear relationship:

$$(C_D)_{ni} = (C_D)_{iso}\,(1 + 0.55\tau) \tag{5}$$

where τ is the nondimensional particle temperature. Thus for τ = unity, Equation (3) would become

$$F_D = (1.55)\ K\,\{(\rho_g)_\infty (C_D)_\infty\} \tag{6}$$

On the other hand, if the mean-film temperature is used for the evaluation of ρ_g and C_D, Equation (3) can be written

$$F_D = K\,\{(\rho_g)_{av}(C_D)_{av}\} \tag{7}$$

Equations (6) and (7) would be compatible if the ratio

$$\{(\rho_g)_{av}(C_D)_{av}\}/\{(\rho_g)_\infty(C_D)_\infty\}$$

had the value of 1.55 for $\tau = 1$.

This ratio was evaluated for nitrogen, using a particle temperature of 600°K and a free stream temperature of 300°K. $(C_D)_{av}$ and $(C_D)_\infty$ were calculated by means of the data of Beard and Pruppacher, using the same relative velocity. The ratio came to 1.34, instead of 1.55, indicating that there is a significant difference between the Kassoy and the mean film temperature approaches. On the other hand, an analogy can be drawn with heat transfer where correction factors of the following type are used to account for the variation of fluid properties with temperature:

$$(N_{Nu})_{ni}/(N_{Nu})_{av} = (\nu_{av}/\nu_{\infty})^n \quad (8)$$

Ahmed (1967) has obtained an experimental value of $n = 0.15$. The following correction was accordingly used to match the two approaches:

$$(C_D)_{ni}/(C_D)_{av} = (\nu_{av}/\nu_{\infty})^n \quad (9)$$

where

$$(C_D)_{ni}/(C_D)_{av} = (1.55/1.34) = 1.16$$

and

$$(\nu_{av}/\nu_{\infty}) = (238/178)(1152/784) = 1.96$$

therefore

$$n = \log(1.16)/\log(1.96) = 0.21$$

However, calculations performed by Hamielec (1971) suggested that only a small correction to the film temperature drag coefficient would be required at a Reynolds number of about 10. (Hamielec also found that natural convection effects would not be expected to have a significant effect for particles of 50 μm diameter.) Thus, contrasting the work of Kassoy et al. (1966) and Hamielec (1971), it is reasonable to suggest that the exponent "n" in Equation (9) varies with Reynolds number from a value of about $n = 0.21$ at $Re < 0.1$ to $n = 0$ at $Re > 10$. Further experimental and computational work is required to confirm this conclusion, but in view of the experimental results of Ahmed (1967) on heat transfer in a Reynolds number regime similar to the present work, an upper limit of about 0.15 should be acceptable.

In a system where the motion in nonsteady (that is, the relative acceleration is nonzero), the flow field around a particle must adapt to the changing relative velocity. As in all real systems there is a relaxation time required for this change to take place. Thus any determination of drag coefficient made within this flow field relaxation time would give a result which would be the sum of a transient and steady state response. In conventional systems, time scales (~1 sec) are much longer than relaxation times (~1 ms) and therefore a transient response is not observed. However, residence times in d.c. plasma flames are of the order of one millisecond and that transient response must therefore be taken into account when considering particle motion.

The full equation of motion for creeping flow (low N_{Re}) as given by Hinze (1959) (and known as the Basset-Boussinesq-Oseen equation) is

$$(\pi/6)\,(D_p)^3(\rho_p)\,(dV_p/dt) =$$
$$- 3\pi(\mu_g)(D_p)(V_p - V_g) + (\pi/6)(D_p)^3(\rho_g)(dV_g/dt)$$
$$- (1/2)(\pi/6)(D_p)^3(\rho_g)\,(dV_p/dt - dV_g/dt)$$
$$- (3/2)(D_p)^2(\pi\mu_g\rho_g)^{1/2} \int_{t_0}^{t} \{(dV_p/dt' - dV_g/dt')$$
$$(t - t')^{-1/2}\}\, dt' + F_e \quad (10)$$

or in a summary form

$$F_P = F_D + F_{PG} + F_{AM} + F_H + F_E \quad (11)$$

Proceeding from left to right, the terms are (i) The particle mass-acceleration product; (ii) The conventional Stokesian drag; (iii) The drag due to the pressure gradient; (iv) The drag due to the so-called added mass; (v) The Basset history term (effectively the transient response mentioned above); (vi) External potential forces (gravitational, electric, magnetic, etc.).

In a plasma jet, the gas density is very low compared to the density of the particles, and therefore the pressure gradient and added mass terms may be neglected. Further, the plasma jet is electrically field-free and therefore there should be no electrical forces on the particles even if they had some charge on them. In addition, gravitational effects are of the order of 0.1% of the steady state drag. The equation of motion would therefore reduce to

$$F_p = F_D + F_H \quad (12)$$

Equation (12) applies to Stokesian motion. On a strict mathematical basis, the equation is only valid for a Reynolds number tending to zero. Odar and Hamilton (1964) investigated the motion of a sphere oscillating in oil at Reynolds numbers up to 62. They claimed that their data could be well represented by using a drag force calculated from the standard drag curve, together with added mass and history terms. However, coefficients in front of these terms were required to account for departure from Stokesian motion. They found that these coefficients appeared to be independent of the Reynolds number but were a function of the reciprocal of the acceleration modulus N_{AM}. An asymptotic value of the history coefficient of 0.48* was suggested for low N_{AM}. More recently, Rimon and Cheng (1969) reported a small but definite relation between Reynolds number and calculated relaxation time. Finally, Odar (1966) noted that the history term is predominant at the very beginning of motion and for the next 10 milliseconds, which is the situation in the present study. His equation also gave a good description of the motion of spheres falling from rest, thus giving it generality.

As a result of the difficulty of applying experimental techniques, very little of a quantitative nature is known about the turbulence characteristics of a plasma jet. Langmuir probe correlation studies have been made at low gas pressures (~10 mmHg) where the indicated turbulence intensity was about 25% (Johnston et al., 1968). It was also shown that the maximum turbulence intensity was not at the centerline, but typically at about eight nozzle diameters off the axis. This is in contrast to free isothermal jets (for example, Abramovich, 1963). Goldschmidt et al. (1971) have recently advanced that there is strong experimental evidence that the turbulence characteristics of the carrier stream are unaffected by the presence of particles if the latter are small and in dilute concentration, which is the situation in the present study. On the other hand, the picture is somewhat complicated by the presence of perturbations of the jet flow field arising from a periodic fluctuation of about 10 kilohertz of the arc discharge (Lewis, 1971), which is peculiar to d.c. plasma devices of the type used in this work.

Studies of the motion of particles in pipe flow (as summarized, for example by Soo, 1967) have generally assumed the Stokesian particle drag coefficient ($C_D = 24/N_{Re}$) to be unchanged by the effect of turbulence. One essential difference between the above studies and the present work is that while the mean velocity of the particles in pipe flow would be generally close to that of the fluid, the mean relative velocity between particle and fluid in a plasma jet would generally be of the order for 100 m/s. In other words, for equal sizes of small particles, the low relative Reynolds numbers attained in the former case are essentially due to the low relative velocities, while in the latter case, they are largely due to the high kinematic viscosity of the medium.

In the absence of information to the contrary, it may

* C_H reported by Odar and Hamilton is actually 6 times this value, because it includes the (3/2) coefficient in the expression for F_H, and the latter is based on the particle radius.

be assumed that the Eulerian scale of turbulence in a plasma jet is of the same order of magnitude as the diameter of the anode-nozzle. In this case, with a 10-mm diameter nozzle and 50-μm particles, the ratio of turbulence length scale to particle diameter would be 200. Because the scale of turbulence is so much larger than the particles and because of the magnitude of the relative velocity, it is visualized that the particle will encounter many different eddies in the course of its residence time in the jet. In view of the complete lack of information, it will be simply assumed for the purpose of this study that each time that a particle is engulfed by a new eddy, it will react as if there had been a change in the free-stream velocity. A Laser-Doppler anemometric study of the turbulence characteristics of plasma jets has been initiated in this laboratory which will hopefully clarify the picture. This information is all the more needed in view of the evidence that turbulence exerts a strong influence on particulate heat transfer (Brandon and Grizzle, 1971). In this connection, the reader is referred to a recent paper by Waldie (1972b) in which he presents a thorough review of the experimental evidence so far published in the field of heat and mass transfer to particles in plasmas.

Noncontinuum effects were studied by Zarin (1970). Increases in drag coefficient in the slip flow regime ($0.01 < Kn < 0.1$) varied between 3 and 8% for Reynolds numbers of 100 and 4,000, respectively. Although the small diameter of the particles used in this study resulted in Knudsen numbers which approached the slip and noncontinuum ranges, these ranges were never entered. For the present work, therefore, these effects may be ignored.

EXPERIMENT

Equipment

The equipment used in this study has been fully described elsewhere (Lewis, 1971; Lewis and Gauvin, 1971; Katta et al., 1973). Only a brief description is given here.

Figure 1 shows the plasma torch, consisting essentially of a thoriated tungsten rod cathode (12-mm diam. with a 45° half-angle point) and a concentric cylindrical copper anode nozzle (inside diam. 10 mm and overall length of 50 mm).

The high-speed camera used for the determination of particle velocities and decelerations was a standard model, operating in streak mode (Lewis and Gauvin, 1971). The streaks produced were the image of a particle tracking at right angles across a film, which was itself moving at constant velocity. Particle velocities were determined by the resultant streak angles and particle decelerations from the change of angles across the film.

The plasma calorimetric probe, used for the determination of both gas temperature and gas velocity, consisted of a triple tube heat exchanger, surrounded by a second concentric exchanger acting as an insulating jacket. The probe was manufactured by the Greyrad Corporation, formerly of Princeton, New Jersey, and is currently being produced by Calprobe, under patent license from J. Grey. Its calibration and use are fully described by Katta et al. (1973).

Experimental Procedure

In brief, the following work was carried out:

1. The torch was operated throughout the experiments in open air with the nozzle pointing vertically downwards, using one fixed set of conditions: Argon plasma gas flow rate: 2.7×10^{-6} m^3/s (35 std. cu. ft./hr.; Arc voltage: 30 V; Arc current: 650 A; and particle carrier gas flow rate (argon): 6.3×10^{-7} m^3/s (8 std. cu. ft./hr.).

2. Axial profiles of temperature and velocity of the argon plasma and concentration of entrained air were determined by use of the calorimetric probe. (During these measurements, no particles were fed into the torch, but the carrier gas was maintained at an equivalent flow rate.) Radial profiles of temperature and velocity were obtained by thermocouple and uncooled Pitot-tube at a distance of 178 mm from the nozzle exit.

3. In turn, five separate fractions of closely-sized glass microspheres (in the overall size range 30 to 140 μm) were injected into the plasma at a rate of about 0.07 g/s. The particles emerging from the plasma flame were photographed at a distance of 178 mm from the nozzle exit, using the high-speed ciné camera operating in streak mode. Analysis of the film records yielded the particle velocities for each group. Only the 44 to 53 μm group record was amenable to determination of particle deceleration (this being due to the small changes of angle for the larger particles and the limiting definitions of the streaks for the smallest particles).

EXPERIMENTAL RESULTS

The basic results are presented in Figures 2 to 5 as follows: Temperature radial profile, Figure 2; Velocity radial profile, Figure 3; Axial profiles of temperature, velocity and concentration, Figure 4; and particle decelera-

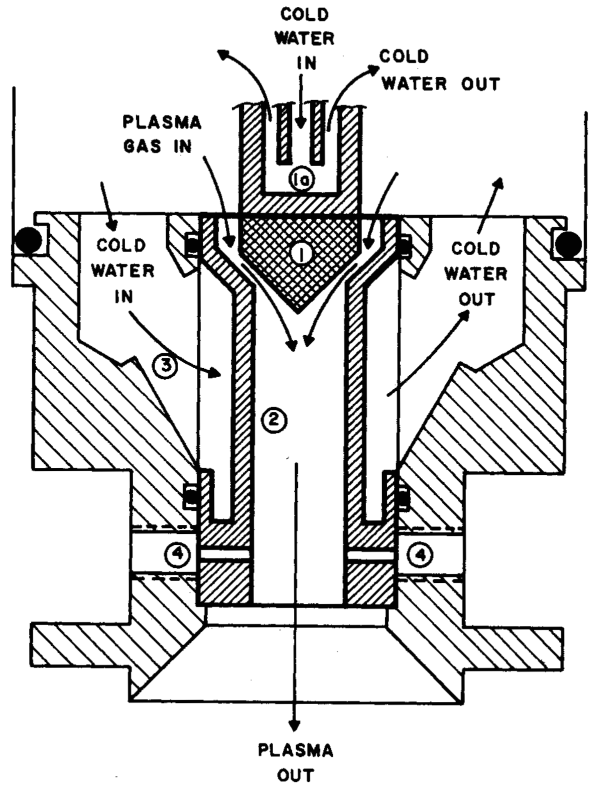

Fig. 1. Cross section of plasma torch: 1. thoriated tungsten cathode; 1a. cathode holder; 2. copper nozzle-anode; 3. nozzle body; 4. particle injection inlet.

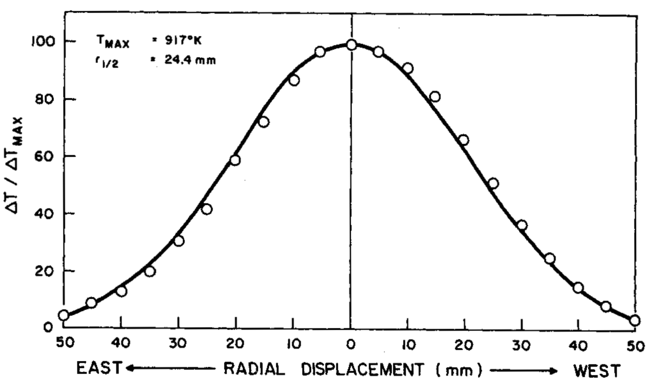

Fig. 2. Radial profile of temperature at 178 mm from torch nozzle exit.

tion versus particle velocity for the 44-53 μm group, Figure 5.

In addition, mean particle axial velocities experimentally obtained are shown as circles on Figure 7. The mean particle velocity decreased from 103 m/s for the 37-μm group to 64 m/s for the 127-μm group. The average ratio of standard deviation to the mean of the particle velocity was about 21%.

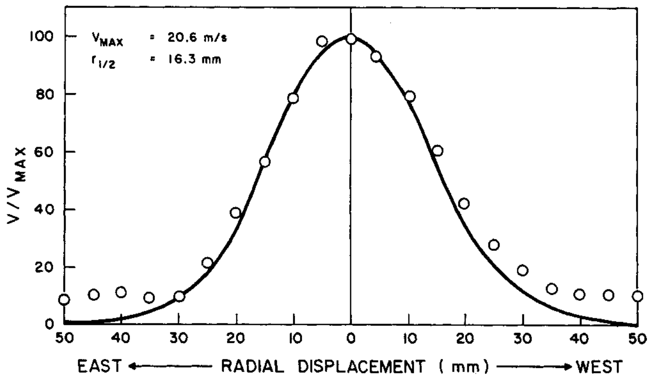

Fig. 3. Radial profile of velocity at 178 mm from nozzle exit.

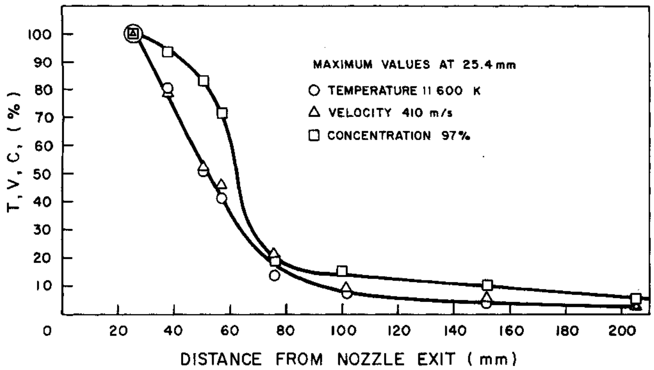

Fig. 4. Normalized axial profiles of temperature, velocity, and concentration (of argon in air).

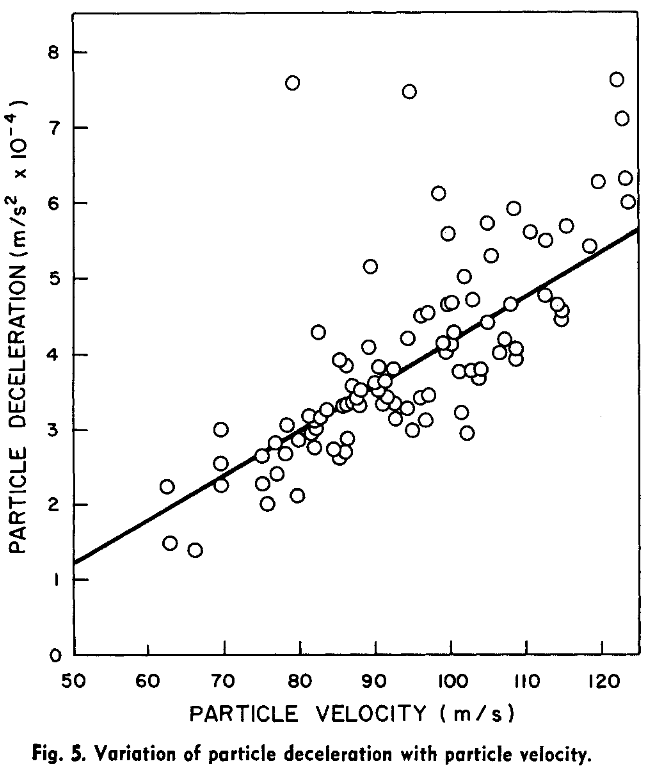

Fig. 5. Variation of particle deceleration with particle velocity.

TABLE 1. EXCESS DRAG: PERCENTAGE INCREASE OF DRAG COEFFICIENT ABOVE THE EXPECTED VALUE

T_p °K n	1600	1900	2200	2500
0	42	43	43	44
0.15	32	29	26	23

For the 44 to 53 μm group, the mean particle velocity was 95 m/s. The least squares linear regression analysis of the deceleration data was

$$-\,dV_p/dt = (1.73 \pm 0.82)(10^4) + (5.85 \pm 0.87)(10^2)(V_p) \quad (13)$$

The particle deceleration corresponding to the mean velocity was $-3{,}900$ g.

The experimental drag coefficient was calculated from the equation:

$$(C_D)_{\text{Exp}} = (-\,4/3)(dV_p/dt)(\rho_p/\rho_g)(D_p)/\{|V_p - V_g|\,(V_p - V_g)\} \quad (14)$$

Because no method was found suitable for measurement, the particle temperature was assumed in turn to be 1,600 K, 1,900 K, 2,200 K, 2,500 K.

The expected nonisothermal drag coefficient was calculated for the equivalent mean film temperature, using the Beard and Pruppacher data (1969) and two kinematic viscosity ratio exponents (0 and 0.15):

$$(C_D)_{ni} = (C_D)_{\text{B+P}}\,(\nu_{av}/\nu_\infty)^n \quad (15)$$

The Excess Drag was then calculated from the defining equation:

$$(C_D)_{xs} = \{(C_D)_{\text{Exp}}/(C_D)_{ni} - 1\}\,(100) \quad (16)$$

The results are summarized in Table 1.

Allowing that the true kinematic viscosity ratio exponent was between 0 and 0.15 and that the true particle temperature was between its boiling point (2,500 K) and the temperature at which it would glow red-hot (1,600 K according to manufacturers), then it is seen that the measured excess drag was somewhere between 23% and 44% or a nominal average of 34%.

SIMULATION

Previous attempts at simulating the motion of particles in plasma flames used rather gross simplifications to obtain residence times. These simplifications included Stokesian motion, isothermal and relaxed flow field (that is, no history), and spherical geometry (when the actual feed was irregular in shape).

In the simulation discussed here, the above factors were taken into account and corrections incorporated into a general computer program [listed by Lewis (1971)]. This program used a Kutta-Merson predictor-corrector process to integrate the equation of motion:

$$(dV_p/dt) = (-\,3/4)(\rho_g/\rho_p)\,\{(C_D)_{ni}/D_p\}\,|V_p - V_g| \ldots (V_p - V_g) - \{(9/\rho_p)(1/\pi)^{1/2}\} \ldots \ldots \int_a^b \{(\mu_{av}\rho_{av})^{1/2}(1/D_p)(dV_p/dt' - dV_g/dt')(t - t')^{-1/2}\}\,dt' \quad (17)$$

The first term on the right represented the Beard and Pruppacher drag term corrected for a nonisothermal flow field using an exponent of 0.15, while the Basset history term was modified to include the $(\mu_{av}\rho_{av})^{1/2}(1/D_p)$ term under the integration sign.

The distribution of temperature, velocity, and concentration of the plasma jet were modeled in the axial direction using the data obtained from the calorimetric probe measurements. While carrying out computations with varying particle diameters, a simple radial profile model was used with the particle radial injection velocity fixed at 10 m/s. (This velocity had been previously determined during actual injection nozzle studies in the cold.) For distances greater than 10 mm beyond the nozzle exit the radial profiles were assumed flat. At the nozzle, the profiles of temperature and velocity were assumed parabolic. While testing the effect of a varying radial injection velocity, a more realistic Gaussian radial profile model was used. Half-radii of temperature and velocity were obtained by interpolation between the data of Carleton (1970) (working close to the nozzle exit of his torch) and the data obtained at a distance of 178 mm from the nozzle exit of the torch used in the present work. According to Kleinstein (1964), the axial profiles of temperature and concentration in a plasma jet should be almost similar. Although not exactly borne out by Figure 4, it was assumed, for simplicity, that the half-radii of temperature and concentration were equal.

In order to calculate mean film temperatures, the particle surface temperature was required. This was calculated using the Ranz-Marshall (1952) correlation corrected for nonisothermal effects (Ahmed, 1967), ionization effects (Petrie, 1969), and recombination effects (Chludzinski, 1964). The latter correction was necessary due to the entrainment of considerable quantities of air by the hot argon jet. At each position of the particle along its trajectory, the magnitude and direction of the relative velocity vector was calculated, followed by the Reynolds number and hence the drag coefficient.

The radial acceleration was taken as the component of the steady state drag in that direction without any history effect. This was justified by preliminary calculations which showed that the history term was generated by the gas deceleration and not the particle acceleration. Since radial velocities in a jet are low, it would be expected that radial history could be ignored.

The axial acceleration of the particle was calculated as the sum of the axial component of the steady state drag and an axial history term evaluated by the method of Odar (1966):

$$\int_a^b \{(\mu_{av}\rho_{av})^{1/2}(1/D_p)(dV_p/dt' - dV_g/dt')$$

$$(t-t')^{-1/2}\}\ dt' \simeq (\overline{\mu_{av}\rho_{av}})^{1/2}(\overline{1/D_p})(\overline{dV_p/dt' - dV_g/dt'})$$

$$\int_a^b (t-t')^{-1/2}\,dt' \simeq (\overline{\mu_{av}\rho_{av}})^{1/2}(\overline{1/D_p})$$

$$(\overline{dV_p/dt' - dV_g/dt'})\ [-2\{t-t'\}^{1/2}]_a^b \quad (18)$$

A set of computations was carried out as above for the mean particle diameter of each of the size ranges used. In addition, a set of computations was made, with the history term neglected.

A third set of results was generated for the 48.5-μm group, for varying particle inlet velocity.

RESULTS OF COMPUTATIONS

The computation results are summarized in Figures 6 to 10. Figure 6 shows typical predictions of particle axial velocity against axial displacement, for 50 and 100-μm particles, with and without tthe history term included. The points of interest are:

1. Without history, the particles accelerated to a maximum velocity where the particle velocity equalled the gas velocity. The particles then decelerated against the braking effect of the gas.
2. With history, the maximum particle velocity occurred prior to the reversal of direction of the relative velocity.

Figure 7 shows the exit axial particle velocities against particle diameter. It is noted that the experimental measurements (also shown) were sandwiched between the predictions with and without history. Thus in the absence of gross computational and experimental errors, it appears that a coefficient should accompany the Basset history term in the equation of motion.

Figure 8 shows computations of particle residence time for an axial distance of 178 mm (with and without his-

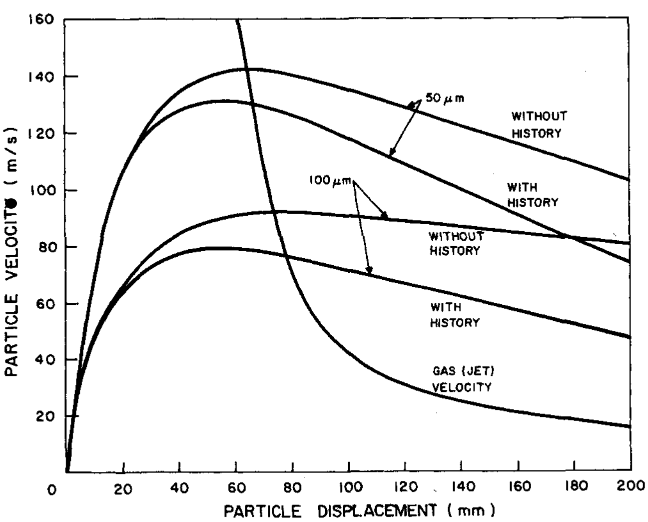

Fig. 6. Predicted axial velocity vs. axial displacement.

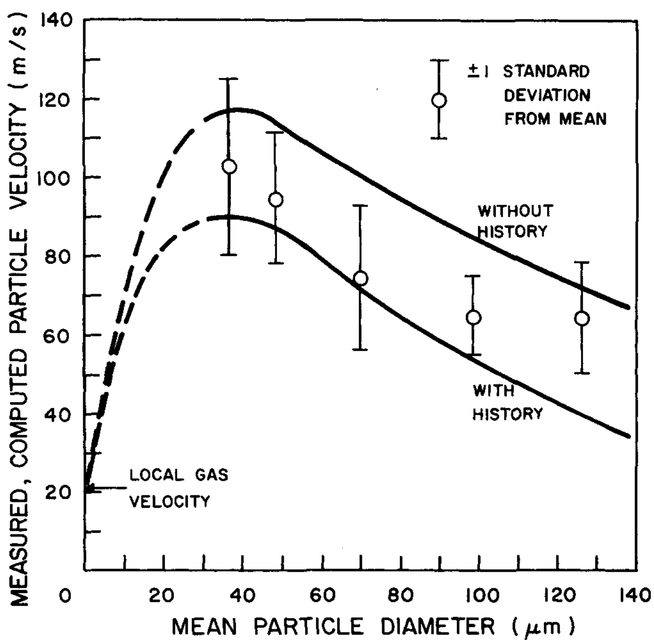

Fig. 7. Measured and predicted axial particle velocity vs. particle diameter.

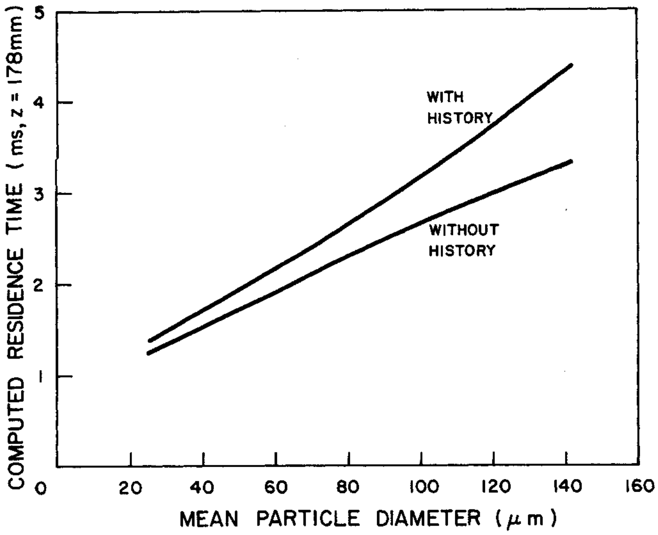

Fig. 8. Computed particle residence time vs. particle diameter.

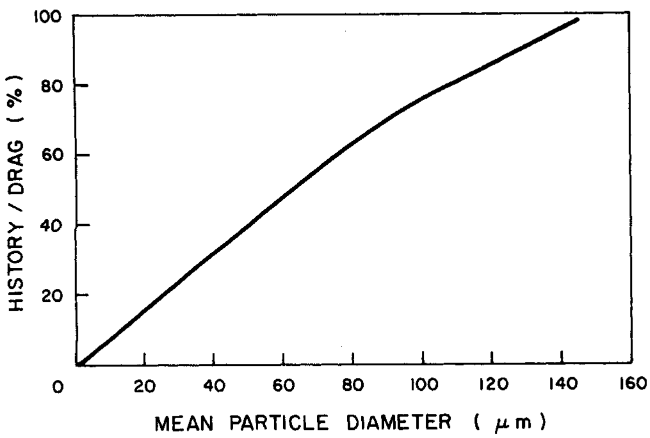

Fig. 9. Computed Basset history effect as a fraction of axial drag.

tory) for varying particle diameters. The trend was from a time of about 1 ms for 30-μm particles to about 4 ms for 140-μm particles. The effect of history was to increase particle residence times by about 10% for 30-μm particles and by about 30% for 140-μm particles.

Figure 9 shows computed history effect (at an axial displacement of 178 mm) as a fraction of the axial drag, ranging from about 25% for 30-μm to 95% for 140-μm particles.

Figure 10 shows the effect of a varying particle radial injection velocity on the computed radial displacement, axial velocity, and residence time for particles having a mean diameter of 48.5 μm. The results showed a maximum predicted particle exit velocity of about 85 m/s with an almost symmetrical fall-off on either side of the inlet velocity of 10 m/s. The minimum computed residence time was about 2 ms for an equivalent radial displacement of 6 mm. (The result of using a more realistic radial profile model of the plasma jet was to reduce the predicted history effect by about 4% with negligible change in the particle exit velocity.)

A special run of the program was made to test the effect of the particles being nonisothermal. For the largest group, the particle surface temperature was assumed to be at its boiling point (2,500 K) during the heating stage and at the temperature of the gas during its cooling stage. No significant difference was found in the predicted particle velocity or acceleration compared to the earlier predictions.

During all the above computations, particle evaporation was predicted to be negligible. The effect of evaporation for the smaller particle diameters of interest was only at the level of the fifth significant figure.

DISCUSSION

Origins of the Distribution of Particle Velocities

The standard deviation of the measured velocities of the five size groups of particles varied from 15% to 25% of the mean velocity, with an average deviation of 21%. The possible origins of these rather wide distributions are discussed below:

1. Measurement errors could have caused a maximum random uncertainty of about 1.5% in the measured particle velocity. [This was discussed in detail in Lewis and Gauvin (1971).]
2. Each group of particles had a size range (±10%)

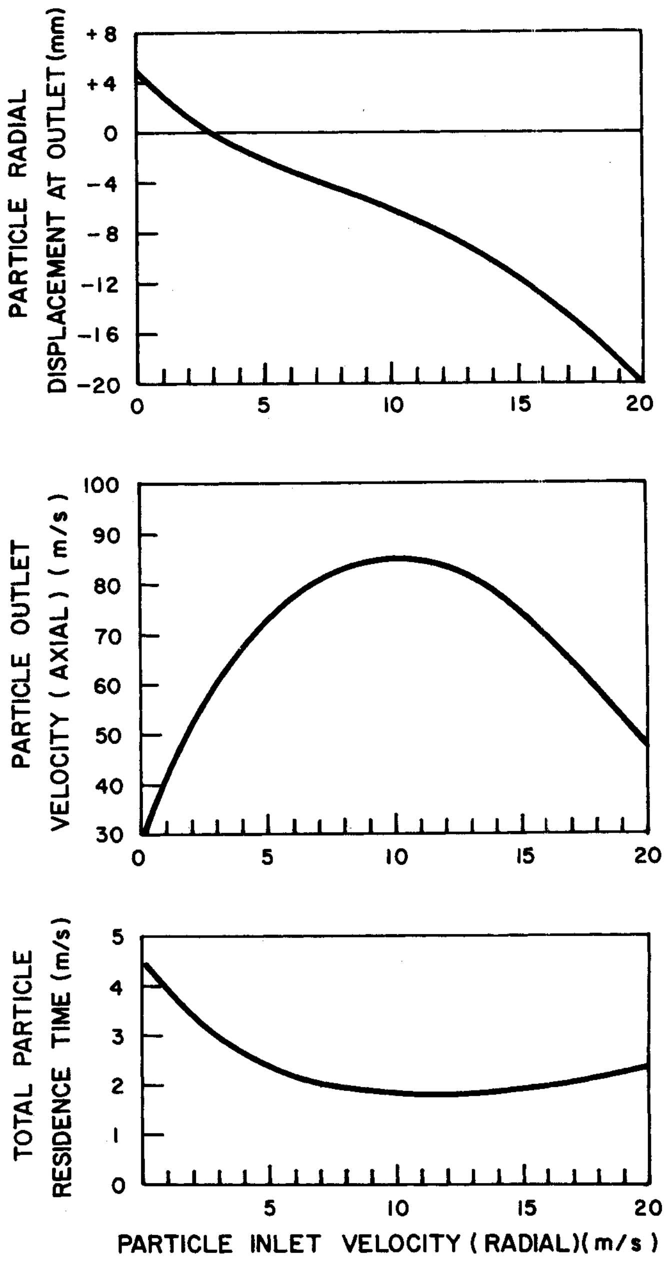

Fig. 10. Effect of variation of particle radial injection velocity on radial displacement, axial velocity, and residence time.

given by the fourth root of two standard sieve sizes. From the computer predictions (Figure 7), it is possible to convert the particle diameter range into a (predicted) velocity range. The error suggested by this method ranged from nearly 0% at $D_p \sim 36\mu m$ to 15% at $D_p \sim 60\mu m$. Thus it can be seen that the range of particle diameters could have contributed significantly to the distribution of particle velocities.

3. It is unrealistic to assume that all particles would enter the plasma jet from the injection nozzle at exactly the same velocity of 10 m/s. Allowing a more than generous variation in this injection velocity between 5 m/s to 15 m/s, the variation in the exit velocity would be about 8% for the 44 to 53 μm group (Figure 10).

4. It can be inferred from an earlier discussion that turbulence should have an important randomizing effect when dealing with the motion of small particles. (The word *turbulence* here includes the perturbations of the jet flow field resulting from the periodic fluctuation of the d.c. arc discharge, as mentioned earlier.) This follows from the fact that, for reasons previously given, a particle injected into the plasma flame will be subject to (say) a fast and hot pocket of gas for an interval of time, while during the next interval it may be subject to a slow and cold eddy. In order to assess the effect of a random component of gas velocity on the particle velocity, a simplified computer simulation was carried out. It was assumed that the particle experienced a local gas velocity everywhere along its trajectory which was only 90% of the measured gas velocity at the particular axial displacement. The predicted particle velocity at the flame exit was found to be reduced from 88.5% m/s to 79.5 m/s (that is, about 10%). This calculation also had the merit of providing an estimate of the possible effect of an error in the gas velocity, resulting from using the large calorimetric probe as a Pitot tube.

In summary therefore, the major causes of particle velocity distribution were probably the range of particle diameters in each size group, the variation in the particle radial injection velocity, and the randomizing effect of the turbulent jet fluctuations.

Effect of the Basset History Term on the Drag Coefficient

Measurement Errors. Lewis and Gauvin (1971) have noted that a random error of about 16.5% could occur in the measurement of the particle deceleration. Since the particle deceleration versus particle velocity data were fitted by a least squares linear regression analysis (correlation coefficient 0.65), it may be suggested that the error in the mean deceleration (calculated from the regression analysis) would not be more than a few percent from the true mean deceleration.

While the random measurement error would in itself explain a large part of the scatter in the deceleration-velocity plot, it would also mask any true variation of particle deceleration at a fixed relative velocity. Potential sources of such variation would be the range of particle diameters ($\pm 10\%$), small variations in particle density, and also possible differences in particle temperature. (The variation in particle temperature could result from the temperature fluctuations of the gas and different particles traveling through different temperature zones of the plasma.)

Simultaneous systematic errors of $\pm 4\%$ and $\pm 9\%$ in the particle velocity and deceleration were also considered possible. To make this point clear a true particle velocity of 104% would be equivalent to a true particle deceleration of 109%. Rewriting Equation (14),

$$C_D = k_D(dV_p/dt)/\{|V_p - V_g|\,(V_p - V_g)\} \quad (19)$$

a simple evaluation of the error in the drag coefficient for a particle velocity of 100 m/s and a gas velocity of 20 m/s would give the ratio of the true to estimated drag coefficient as*

$$(C_D'/C_D) = \{109/(104 - 20)^2\}/\{100/(100 - 20)^2\} = 0.99 \quad (20)$$

Thus the measured drag coefficient would be underestimated by 1%. The error in the Reynolds number would be given by

$$(Re'/Re) = (104 - 20)/(100 - 20) = 1.05 \quad (21)$$

Assuming for simplicity, that the motion was near Stokesian, the error in the expected drag coefficient would be

$$(C_D'/C_D) = 1/1.05 = 0.95 \quad (22)$$

Thus the error in the excess drag would be

$$\{(0.99/0.95) - 1\}\,(100) = 4\% \quad (23)$$

It was also noted (Lewis and Gauvin, 1971) that the radial displacement of particles could be as much as 5 mm from the axis. This was confirmed by the computer predictions, which indicated a possible radial displacement of particles of about 6 mm. The error in the relative velocity due to the Gaussian distribution would be about 1% as seen below:

$$(V_p - V_g)'/(V_p - V_g) = \{(100 - 0.95 \times 20)/(100 - 20) - 1\}\,(100) = 1.25\% \quad (24)$$

Thus in summary, the systematic and random errors of measurement would lead to errors of a few percent in the estimation of the excess drag. Compared to the question of the effects of particle temperature and kinematic viscosity ratio exponent, these systematic and random error effects are not considered to be of major importance.

Comparison of Measured Excess Drag and the Computed History Effect. The predicted history effect for the 44 to 54 μm group of particles varied between 28% and 36%, depending on the radial injection velocity and the simulation of the radial profile. The lower value was considered to be the more reasonable value since it corresponded to more realistic values of these two parameters. These predictions should be compared with the experimental excess drag (24 to 44%). In view of the experimental difficulties and the uncertainties in the simulation, the agreement between prediction and measurement is considered quite good. Therefore it is reasonable to conclude that a coefficient should accompany the Basset history term in the equation of motion for calculations where the particle Reynolds number changes rapidly with time, as pointed out by Odar and Hamilton (1964) and by Odar (1966). The physical significance of this coefficient is not clear. The above mentioned authors have found that its value depended on the Acceleration number (the reciprocal of the acceleration modulus, N_{AM}) and had a value of unity when the acceleration number was zero, that is, when the forces due to the local acceleration largely predominated over those due to convective acceleration.

Comparing the predictions of particle velocities both with and without history, one can see that, as a first ap-

* Primed quantities refer to true values, unprimed quantities refer to the measured values.

proximation, the use of Odar's coefficient of 0.48 would have given a prediction of particle velocity which would agree fairly closely with the experimental results.

NOTATION

C_D = drag coefficient
$(C_D)_{av}$ = drag coefficient evaluated at mean film temperature
$(C_D)_{B+P}$ = Beard and Pruppacher drag coefficient
$(C_D)_{Exp}$ = experimental drag coefficient
$(C_D)_{iso}$ = isothermal drag coefficient
$(C_D)_{ni}$ = nonisothermal drag coefficient
$(C_D)_{xs}$ = excess in experimental drag coefficient, Equation (16)
D_p = particle diameter
F_D = drag force on particle
F_E = force due to potential field
F_H = force due to Basset history term
F_p = total force on particle
F_{PG} = force due to pressure gradient
K = defined in Equation (3)
k_D = defined in Equation (19)
n = kinematic viscosity ratio exponent
N_{Nu} = Nusselt number
$(N_{Nu})_{ni}$ = nonisothermal Nusselt number
$(N_{Nu})_{av}$ = Nusselt number evaluated at mean film temperature
N_{AM} = acceleration modulus,

$$|dV_p/dt - dV_g/dt|\ (D_p)/(V_p - V_g)^2$$

$r_{1/2}$ = half-radius
N_{Re} = Reynolds number (evaluated at mean film temperature)
T = temperature
T_p = particle temperature
T_∞ = bulk gas temperature
t = real time
t' = convolution time (for history integral)
V_g = gas velocity
V_p = particle velocity
μ_g = viscosity of gas
$(\mu_g)_{av}$ = viscosity of gas evaluated at mean film temperature
ν = kinematic viscosity of fluid
ν_{av} = kinematic viscosity evaluated at mean film temperature
ν_∞ = kinematic viscosity evaluated at bulk gas temperature
ρ_g = gas density
$(\rho_g)_{av}$ = gas density evaluated at mean film temperature
$(\rho_g)_\infty$ = gas density evaluated at bulk gas temperature
τ = temperature ratio $(T_p - T_\infty)/(T_\infty)$

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